

Combinatorics

2018. April

1. Prove that triangle-free planar graphs are 3-degenerate.
2. Find the formula for Fibonacci number F_n , which is defined by the recurrence $F_n = F_{n-1} + F_{n-2}$ (for $n \geq 2$) and $F_0 = 1, F_1 = 1$.
3. Consider a triangle array of $\binom{n+1}{2}$ balls, with 3 colors. Two array are considered equal if one can be transformed into the other by a rotation (with angle $2\pi/3$ or $4\pi/3$). Count the number of such triangle arrays.
4. Prove that $\sum_{k=0}^n \binom{s+k}{k} \binom{n-k}{m} = \binom{s+n+1}{s+m+1}$.
5. Prove that the number of partitions of n into distinct parts is equal to the number of partitions of n into odd parts. For example $n = 7$, the partitions are 7, 61, 52, 43, 421 and 7, 511, 331, 31111, 1111111 respectively.