Effective Condition Number for Weighted Linear Least Squares Problems and Applications to the Trefftz Methods

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Abstract

In this talk, we extend the effective condition number for weighted linear least squares problem with both full rank and rank-deficient cases. First we apply the effective condition number for the linear algebraic equations of the notorious Hilbert matrix, $\mathbf{H} \in \mathbf{R}^{n\times n}$. The traditional condition number is huge for not small *n*, but the effective condition number is small. When $n = 10$, $\text{Cond} = 0.16 \times 10^{14}$. On the other hand, for 10^8 number of random right hand vectors, the maximal effective condition number is less than 10^5. Furthermore, we apply the effective condition number to the collocation Trefftz method (CTM) [1] for Laplace's equation with a crack singularity, to prove that $\text{Cond\_eff} = O(\sqrt{L})$ and $\text{Cond} = O(L^{\frac{1}{2}}(\sqrt{2})^L)$, where *L* is the number of singular particular solutions used. The Cond grows exponentially as *L* increases, but Cond_eff is only $O(\sqrt{L})$. The small effective condition number explains well the high accuracy of the TM solution, but the huge Cond can not.

Keywords: Condition number, effective condition number, perturbation, weighted linear least
squares problem, collocation Trefftz method, singularity problem.

Reference

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