Meshfree Methods by Weakened Weak (W2) Formulations

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Abstract: This paper provides a brief overview on meshfree methods. We introduces first the fundamental theory for general numerical methods, the G space theory that allows the use of much more types of methods/techniques to create shape functions for numerical methods. Weakened weak (W2) formulations can then be used to construct many meshfree methods. We prove that the numerical methods developed based on the W2 formulation will be spatially stable, and convergent to exact solutions. We next present examples of some of the possible W2 models, and show the major properties of these models: 1) it is variationally consistent in a conventional sense, if the solution is sought from a proper H space (compatible cases); 2) it passes the standard patch test when the solution is sought in a G space with discontinuous functions (incompatible cases); 3) the stiffness of the discretized model is reduced compared to the FEM model and even the exact model, allowing us to obtain upper bound solutions with respect to both the FEM and the exact solutions; 4) the W2 models are less sensitive to the quality of the mesh, and triangular meshes can be used without any accuracy problems. These properties and theories have been confirmed numerically via examples.

Keywords: Numerical methods, meshfree methods, FEM, real-time computation, solution bound, inverse analysis.