A quadrature rule for numerical integration based on Haar wavelets and hybrid functions for multi-dimensional highly oscillatory integrals

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Abstract
In this paper Haar wavelets and hybrid functions have been applied for numerical solution of multi-dimensional highly oscillatory integrals with variable and constant limits of integration. The new approach has two major advantages over the classical methods based on quadrature rule: (i) No need of finding optimum weights as the wavelet coefficients serve the purpose of optimal weights automatically (ii) Mesh points of the wavelets algorithm are used as nodal values instead of considering the $n$ nodes as unknown roots of polynomial of degree $n$. The new method is simple, more efficient and numerically stable. The novel method is compared with existing methods and applied to a number of benchmark problems. Accuracy of the method is measured in terms of absolute errors.

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