Flexible local approximations using fundamental solutions

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Traditional finite difference schemes rely on a structured grid of points in order to define the difference operator. To overcome the need for a structured grid, some non-standard approaches have been proposed, which attempt to generalize the distribution of points by relaxing the grid requirements, thus allowing for difference operators to be obtained on non-structured distribution of points.

One of those non-standard approaches, the difference schemes with flexible local approximations proposed in (Tsukerman, I 2006), resorts to Trefftz-type functions to construct finite difference operators that are free from any form of structured grid.

The same essential characteristic of the Trefftz functions, which is that of being actual solutions of the homogeneous equation for a given problem, is shared by the fundamental solutions, the building blocks of the boundary element method and of the method of fundamental solutions, traditionally used to build boundary-only global approximations in the domain of interest.

Given the shared features of both Trefftz functions and fundamental solutions it seems appropriate to build local approximations, in the way proposed by Tsukerman, whereby the Trefftz functions are replaced by fundamental solutions.

The use of fundamental solutions in a local framework guarantees accuracy while simultaneously eliminating (or strongly reducing) potential problems due to numerical ill-conditioning that are normally present in boundary-only solution methods.

Applications to two-dimensional potential problems (both Laplace and Poisson) are considered.

Comparison with other results available in the literature shows that the method is accurate, reliable and may be considered to be an alternative to other numerical methods.

The generalization of the grids allows for changing the grid in a quite effective manner: adding patches of grid points in any shape or size or density on different domains is now simplified. This is a good indication on the capabilities of the method for dealing with complex geometries especially when coupled with domain decomposition techniques.

Tsukerman I. A class of difference schemes with flexible local approximation. Journal of Computational Physics 2006; 211:659–699