Multiquadric and Its Shape Parameter

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Abstract

Hardy's multiquadric and its related interpolators have been found to be highly efficient for interpolating continuous, multivariate functions, as well as for the solution of partial differential equations. Particularly, the interpolation error can be dramatically reduced by varying the shape parameter to make the interpolator optimally flat. This improvement of accuracy is accomplished without reducing the fill distance of collocation points, that is, without the increase of computational cost. There exist a number of mathematical theories investigating the multiquadric family of radial basis functions. These theories are often not fully tested due to the computation difficulty associated with the ill-conditioning of the interpolation matrix. This paper overcomes this difficulty by utilizing arbitrary precision arithmetic in the computation. The issues investigated include conditional positive definiteness, error estimate, optimal shape parameter, traditional and effective condition numbers, round-off error, derivatives of interpolator, and the edge effect of interpolation.