Coupling three-field and meshless mixed Galerkin methods using radial basis function to solve parabolic equation

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Abstract

In this work we extend the method based on the coupled meshless mixed Galerkin approximation using radial basis function with the three-field domain decomposition method to solve parabol equation. Numerical results of transport-diffusion flow in porous media are given.

Introduction

In recent years, the radial basis functions(RBFs) have seen an increased interest in their use for solving partial differential equations(PDEs). This approach is very attractive due to the fact that it is a truly meshless method and spatial dimension independent, which can easily be extended to solve high dimensional problems. Furthermore, since the RBFs are smooth, it can easily be applied to solve high order differential equations. The technique has been used under collocation and Galerkin formulation [1-4]. The application of radial basis functions as interpolation functions has one major disadvantage which is the large condition number of algebraic system. To deals with this problem domain decomposition is applied considerably. The idea behind the domain decomposition technique is to divide the domain into a number of subdomains and solve the problem as a series of sub-problems [5,6].

The main objective of this paper is to extend the coupled three-field domain decomposition method with mixed Galerkin formulation based on radial basis functions, formulated by Fili et al in [1,2], to solve diffusion-convection problems. To demonstrate the accuracy and the efficiency of the proposed technique we applied it to an example with known analytical solution and compare the results obtained to those of finite volume method.

Numerical results

The approach has been applied to transport problem given by the following equation

$$\begin{cases} \dfrac{\partial c}{\partial t} - div(\nabla c) = f(x,y,t) & \text{in} \quad \Omega\times]0,\ T[\\ c = g & \text{on} \quad \partial\Omega\times]0,\ T[\\ c(.,0) = c_0 & \text{in} \quad \Omega \end{cases} \tag{1}$$

Using $f(x,y,t) = x(x-1)y(y-1) - 2(t+1)(x(x-1)+y(y-1))$, $(x,y) \in \Omega$ and $t \in]0\ T[$, the problem has the following analytic solution $c(x,y,t) = (1+t)x(x-1)y(1-y)$.
To show the efficiency of the proposed technique, we first solve the problem using finite volume method (FVM) with a vertex center scheme and using 1462 elements with 782 nodes. The computing relative error at different time steps is shown in table 1. Then, we solve the same problem by by the coupled mixed Galerkin method with the three-field domain decomposition method (MG-TFDD). The domain was divided into 4 sub-domains with the 194 centers distributed on the four domains. Table 2 show the error at different time steps using MG-TFDD.

t	0.1	0.25	0.5	1
$\|\|u_t - u_{h,t}\|\|_{L^2}/\|\|u_t\|\|_{L^2}$	3.2e-3	3.2e-3	3.2e-3	3.2e-3

Table 1: Results obtained using coupled three-field decomposition with mixed Galerkin method.

t	0.1	0.25	0.5	1
$\|\|u_t - u_{h,t}\|\|_{L^2}/\|\|u_t\|\|_{L^2}$	0.0902627	0.0762057	0.17912	0.378118

Table 2: Results obtained using finite volume method of vertex center type

From table 1 and 2 we can observe that the relative error obtained by the coupled three-field domain decomposition with mixed Galerkin method is stable during time compared to that obtained by finite volume method. It can also be remarked that although the number of nodes used by the MG-TFDD is smaller then that of finite volume method, the relative error for the proposed method is smaller compared to the one obtained by FVM.

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