Hybrid Finite Elements for Strain Gradient Elasticity: Theory and Patch Tests

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The paper presents an application of the strain gradient theory to two-dimensional and three-dimensional hybrid finite elements. The implemented strain gradient theory, as proposed by Aifantis, is a "weakly" nonlocal elasticity model for homogeneous materials based on one additional parameter (besides the Lamé's constants): the "material characteristic length" g^2. It is shown that the hybrid finite element formulation – as proposed by Pian and generalized by Dumont for finite and boundary elements – provides a natural conceptual framework to properly deal with the interelement compatibility of the normal displacement gradients, in which “corner nodes” are not an issue. The theoretical formulation leads to the definition of static boundary conditions interpreted as normal and tangential double forces, besides the boundary forces of the classical elasticity. General families of 2D and 3D finite elements are presented. The consistency of the formulation is assessed by means of generalized patch tests and in terms of the spectral properties of the resultant matrices for linear, quadratic and cubic 2D elements. Moreover, convergence tests are performed for some irregularly-shaped numerical models.

Keywords: Gradient elasticity, variational methods, hybrid finite element, Hellinger-Reissner potential, patch tests, spectral properties.