Indirect Trefftz method in the non-stationary problems

Krzysztof Grysa

Faculty of Management and Comp. Modeling., Kielce University of Technology
Al. 1000-lecia P.P. 7, 25-314 Kielce
Poland
krzysztof@grysa.pl, grysa@tu.kielce.pl

Trefftz method has been known since the work of Trefftz in 1926. Thereafter the method has been extensively developed in both formulation and application. For a long time the Trefftz method was applied only to stationary problems or to problems in which authors used the method to get rid of time. In late nineties Trefftz method was broadened for the non-stationary problems.

Here the indirect Trefftz method is considered. In such approach, an approximate solution of the initial-boundary problem is a linear combination of Trefftz functions. The unknown coefficients are determined from the conditions of approximate fulfilling the boundary and initial conditions. In the case of inverse problems certain additional conditions are included (e.g. measured or anticipated values of searched solution inside the considered body).

In order to find the approximate solution of a direct or inverse problem frequently the FEM is applied Such approach leads to the so called FEMT (finite element method with Trefftz functions), for stationary problems widely described in the papers of Jiroušek, Herrera, and others. In the case of inverse problems their ill-posedness requires additional terms in an objective functional, type of regularization or penalty terms.

Here some remarks concerning the Trefftz indirect method referred to the non-stationary inverse problems are presented. Certain results for non-stationary problems of heat conduction (among others boundary temperature identification and thermal diffusivity estimation), for beam vibration, for elastokinetics and for wave equation (direct and inverse problem of membrane vibrations) are presented. Three kinds of FEMT are tested for direct and inverse non-stationary problems: (a) FEMT with the condition of continuity of temperature in the common nodes of elements, (b) no temperature continuity at any point between elements and (c) nodeless FEMT (substructuring).

An idea of physical regularization of noisy input data, based on Trefftz function, is discussed. In the problem of membrane vibration the approximate solutions of both, direct and inverse problem, even for slightly disturbed input data ((N(0, 0.01)) are completely wrong. However, an error of approximate solution obtained with the noisy data smoothed with the use of Trefftz functions practically does not exceed the inaccuracy of the input data.

Trefftz functions for linear partial differential equations are useful also for problems with a heat source moving on a boundary.

Keywords: indirect Trefftz method, non-stationary inverse problems, FEMT, input data smoothing, heat conduction, elastokinetics, vibrations