Application of the Method of Fundamental Solutions for Inverse Problem of Determination of the Biot Number

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Several functions and parameters can be estimated from the inverse heat conduction problem : static and moving heating sources, material properties, initial conditions, boundary conditions, etc.. This study is limited to the estimation of the heat transfer coefficient for steady state heat conduction problem. This problem is one of the versions of so called Robin inverse problem. Several numerical methods have been proposed for solving the Robin inverse problem, in particular in the context of corrosion detection [5, 8]. Among these methods the most popular one is the boundary element method [1,3,7,9,10]. Other methods used are Galerkin method [4, 5], finite element method [8, 11], control volume method [2]. The authors usually use the Cauchy data on some part of boundary [2-8] as an additional condition. In the paper [1] the author uses the inner temperature for the identification of the heat transfer coefficient that is a function of the boundary coordinates.

In this paper the identification of the Biot number is carried out based on the boundary data and knowledge of temperature in some points inside the domain. The method of fundamental solution (MFS) is proposed to solve 2-D inverse problem of determination of the heat transfer coefficient (dimensionless Biot numbers).As of right now, the MFS was applied in inverse heat conduction problems involving the identification of heat sources, boundary heat flux, Cauchy problem, or boundary determination problem. To the best knowledge of the authors, this paper is a first application of this method to the inverse heat conduction concerned with the identification of the Biot number.

The paper deals with the iterative inverse determination of the Biot numbers for a 2-D steady-state heat conduction problem. The MFS is used to solve the 2-D dimensionless heat conduction problem

$$\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}=0, \text{ for } \Omega=[0,1]\times[0,1], \tag{1}$$

with boundary conditions:

$$\begin{aligned}\theta(X_b,Y_b)&=\theta_b, \quad \text{, on } \partial\Omega_1,\\ \frac{\partial\theta(X_b,Y_b)}{\partial n}&=q_b, \qquad \text{on } \partial\Omega_2,\\ \frac{\partial\theta(X_b,Y_b)}{\partial n}&=-Bi\cdot\theta(X_b,Y_b), \qquad \text{on } \partial\Omega_3,\end{aligned} \tag{2}$$

where θ is the temperature field, θ_b is dimensionless temperature on the boundary $\partial\Omega_1$, q_b is dimensionless heat flux on the boundary $\partial\Omega_2$, $Bi = \frac{\alpha l}{\lambda}$ is unknown Biot number. Using the MFS the solution of equation (1) has a form:

$$\theta(X,Y) = \sum_{j=1}^{M} C_j \ln\left(r_j^2\right) \tag{3}$$

The identification of the unknown value of coefficients C_j of the solution (3) and the identification the value of the Biot number is obtained from collocation of the boundary conditions (2) and additionally from collocation of data from the knowledge of temperature inside the domain:

$$\theta\left(X_i, Y_i\right) = \theta_i\,,\quad i = 1,2,...,N\,, \tag{4}$$

For substitution $C_{M+1} = Bi$ the non-linear system of equation *r* obtained from collocation of the boundary (2) and the additional collocation conditions (3) can be solved in least square sense:

$$E\left(C_1,\ldots,C_{M+1}\right) = \sum_{i=1}^{NG}\left[r_i\left(C_1,\ldots,C_{M+1}\right)\right]^2,$$

$$\frac{\partial E}{\partial C_k} = 2\sum_{i=1}^{NG}\left[r_i\left(C_1,\ldots,C_{M+1}\right)\right]\frac{\partial r_i}{\partial C_k} = 0, \qquad k = 1,2,..,M+1. \tag{5}$$

using the Levenberg-Marquadt iteration method:

$$\sum_{l=1}^{M+1} A'(k,l)\cdot \delta C(l) = b(k), \qquad k = 1,2,..,M+1 \tag{6}$$

where $b(k) = -\frac{\partial E}{\partial C_k}$, $A(k,l) = \frac{\partial^2 E}{\partial C_k \partial C_l}$, $A'(k,l) = \begin{cases} A(k,l)\cdot(1+\lambda) & k = l \\ A(k,l) & k \neq l \end{cases}$.

The accuracy of the proposed method is tested for several different examples in which the the Biot number is described as a constant $Bi = const.$, as a function of coordinate of boundary $Bi = \sum_{j=1}^{J} Bi_j X^{j-1}$ or as a function of temperature $Bi = \sum_{k=1}^{K} Bi_k T^{k-1}$.

The influence of measurement errors on the Biot number and on the identification of temperature field were also investigated l.

Keywords: Method of Fundamental Solutions, Inverse Problem, Levenberg-Marquadt method, the Biot Number

References:

[1] D Maillet, A. Degiovanni, R. Pasquetti, Inverse Heat Conduction Applied to the Measurement of Heat Transfer Coefficient on a Cylinder: Comparison Between an Analytical and Boundary Element Technique, Journal of Heat Transfer, 113 (1991) 549-557

[2] J. Xu, T. Chen, A Nonlinear Solution of Inverse Heat Conduction Problem for Obtaining the Inner Heat Transfer Coefficient, Heat Transfer Engineering, 19(2) (1998) 45-53.

[3] T. J. Martin, G. S. Dulikravich, Inverse Determination of Steady Heat Convection Coefficient Distributions, Journal of Heat Transfer, 120 (1998) 328-334.

[4] D. Fasino, G. Inglese, Discrete methods in the study of an inverse problem for Laplace's equation, IMA Journal of Numerical Analysis 19 (1999), 105-118

[5] D. Fasino, G. Inglese, An inverse Robin problem for Laplace’s equation: theoretical results and numerical Method, Inverse Problems 15 (1999) 41–48

[6] S. Chantasiriwan, Inverse Determination of Steady-State Heat Transfer coefficient, Int. Comm. Heat Mass Transfer. 27(8) (2000) 1155-1164

[7] W. Fang, M. Lu, A fast collocation method for an inverse boundary value problem, International Journal For Numerical Methods In Engineering, 59 (2004) 1563–1585.

[8] M. Slodička, R.Van Keer, A numerical approach for the determination of a missing boundary data in elliptic problems, Applied Mathematics and Computation, 147(2004) 569-580.

[9] F. Lin, W. Fang, A linear integral equation approach to the Robin inverse problem, Inverse Problems 21 (2005) 1757–1772

[10] B.Jin, Conjugate gradient method for the Robin inverse problem associated with the Laplace equation, International Journal for Numerical Methods in Engineering, 71 (2007) 433–453.

[11] B.Jin, J. Zou, Inversion of Robin coefficient by a spectral stochastic finite element approach, Journal of computational Physics 227 (2008) 3282-3306.