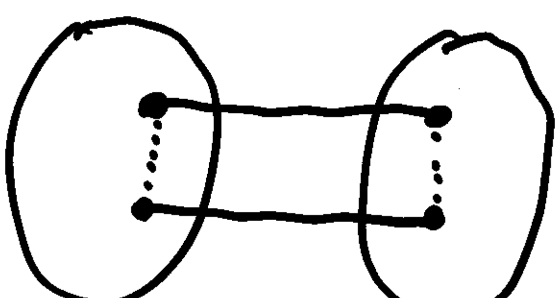
Open Problems

(1) Graphs $\rightsquigarrow$ matchings

* 1-factorable Conjecture:

Every r-regular simple graph of order $2n$ with $r \geq n$, is r-edge-colorable.
has an r-edge-coloring
is 1-factorable

• The bound "$r \geq n$" is sharp.

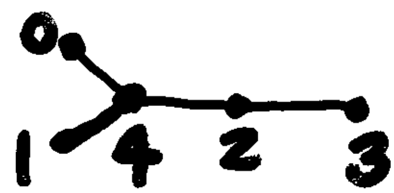


$K_{2m+1} - e$ $\quad$ $K_{2m+1} - e$

(2) Graphs $\rightsquigarrow$ trees

* Ringel conjectures

(i) A tree T of size m is said to be <u>graceful</u> if there is an onto mapping $f : V(T) \longrightarrow \{0, 1, 2, \cdots, m\}$ such that $\{|f(x) - f(y)| : xy \in E(T)\} = \{1, 2, 3, \cdots, m\}$.



Conjecture:
Every tree is graceful.

(ii)

Conjecture:

Suppose T is a tree of size m.

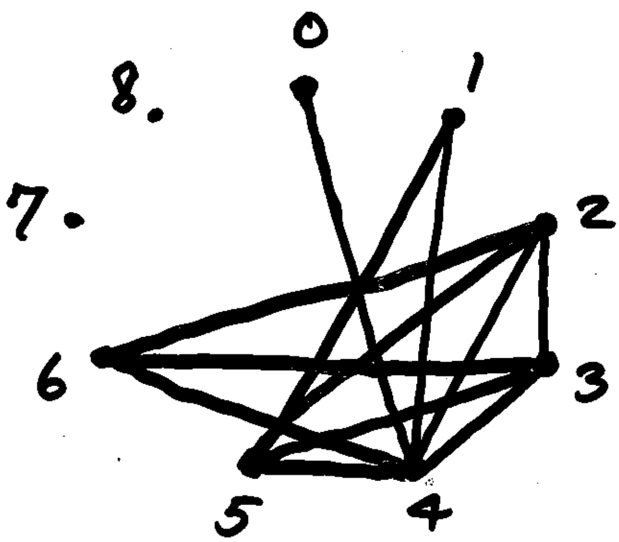
Then K_{2m+1} is T-decomposable, i.e., $T \mid K_{2m+1}$.

- Theorem. (Rosa)

If T is a graceful tree of size m.

Then $T \mid K_{2m+1}$.

- **Idea**



* ASD Conjecture for trees:

Given n trees $T_1 \subseteq T_2 \subseteq \cdots \subseteq T_n$ with $|E(T_i)| = i$, $i = 1, 2, \cdots, n$, then K_{n+1} has an ASD with members $T_1, T_2, \cdots, T_n$.

(3) Graphs $\rightsquigarrow$ paths

The <u>path number</u> $p(G)$ of a simple graph G is the minimum number of paths needed to partition $E(G)$.

Conjecture: $p(G) \leq \left\lceil \frac{|V(G)|}{2} \right\rceil$

- The bound is sharp.

$$p(K_{2n+1}) = n+1$$