Adaptive choosability of planar graphs with sparse short cycles

Albert Guan and Xuding Zhu*
Department of Applied Mathematics
National Sun Yat-sen University
Kaohsiung, Taiwan
National Center for Theoretical Sciences, Taiwan.
E-mail address: zhu@math.nsysu.edu.tw

Abstract

Given a (possibly improper) edge-colouring F of a graph G, a vertex colouring of G is *adapted to* F if no colour appears at the same time on an edge and on its two endpoints. A graph G is called *adaptably* k*-choosable* (for some positive integer k) if for any list assignment L to the vertices of G, with $|L(v)| \geq k$ for all v, and any edge-colouring F of G, G admits a colouring c adapted to F where $c(v) \in L(v)$ for all v. This paper proves that a planar graph G is adaptably 3-choosable if one of the following holds: (1) Any two triangles in G do not intersect and no triangle is adjacent to a 4-cycle or a 5-cycle. Moreover, any 6-cycle is adjacent to at most two triangles. (2) Any two triangles in G have distance at least 2 and no triangle is adjacent to a 4-cycle. (3) Any two triangles in G have distance at least 2 and no triangle is adjacent to a 5-cycle.

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1 Introduction

Let G be a multigraph (i.e., multiple edges are allowed), and let $F : E(G) \to \mathbb{N}$ be a (possibly improper) colouring of the edges of G. A k-colouring $c : V(G) \to$

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$\{1, \ldots, k\}$ of the vertices of G is *adapted* to F if for every $uv \in E(G)$, $c(u) \neq c(v)$ or $c(v) \neq F(uv)$. In other words, the same colour never appears on an edge and both its endpoints. If there is an integer k such that for any edge colouring F of G, there exists a vertex k-colouring of G adapted to F, we say that G is *adaptably k-colourable.* The smallest k such that G is adaptably k-colourable is called the *adaptable chromatic number* of G, denoted by $\chi_{ad}(G)$.

The concept of adapted colouring of a graph was introduced by Hell and Zhu in [9], and has connections with matrix partition of graphs, graph homomorphisms, and full constraint satisfaction problems [3, 4, 5, 8].

Let $L : V(G) \rightarrow 2^{\mathbb{N}}$ be a list assignment to the vertices of a graph G, and F be a (possibly improper) edge colouring of G. A colouring c of G adapted to F is an *L-colouring adapted to F* if for any vertex $v \in V(G)$, we have $c(v) \in L(v)$. If for any edge colouring F of G and any list assignment L with $|L(v)| \geq k$ for all $v \in V(G)$ there exists an L-colouring of G adapted to F, we say that G is *adaptably k-choosable.* The smallest k such that G is adaptably k-choosable is called the *adaptable choice number* of G, denoted by $\text{ch}_{ad}(G)$. The concept of adapted list colouring of graphs and hypergraphs was introduced by Kostochka and Zhu in [10].

Adapted list colouring can be used as a model for scheduling problems. Compared to the original list colouring model, the adapted list colouring allow different constraints for different colours. For example, suppose there is a set of basketball games that need to be scheduled into a set of time slots. The time slots are the colours. The constraints are (1): each game has a list of permissible time slots, and (2): some pairs of games cannot be assigned to the same time slot. This problem is modeled as a list colouring problem. It may happen that two games a, b cannot be both assigned to time slot i, however, they can be both assigned to time slot j. The adapted list colouring of graphs provides a model for this problem.

In ordinary colouring of a graph, parallel edges can be replaced by a single edge. However, for adapted colouring, parallel edges can have different colours and hence cannot be replaced by a single edge. Indeed, if G is a simple graph, and G' is obtained from G by replacing each edge of G with k parallel edges with colours $1, 2, \cdots, k$, then G is k-colourable if and only if G' is adaptably k-colourable.

In this paper, we shall be interested in adaptable colouring of simple graphs. In the remainder of the paper, all graphs are assumed to be simple.

Since a proper vertex k-colouring of a graph G is adapted to any edge colouring of G, we have $\chi_{ad}(G) \leq \chi(G)$ and $\text{ch}_{ad}(G) \leq \text{ch}(G)$ for any graph G, where $\chi(G)$ is the usual chromatic number of G, and $\text{ch}(G)$ is the usual choice number of G. In particular, if G is a planar graph, then $\chi_{ad}(G) \leq 4$. This bound turns out to be sharp. It was shown in [9] that there exist simple planar graphs that are not adaptably 3-colourable.

The adaptable choosability of planar graphs was studied in [2]. It was proved in [2] that

- Planar graphs are adaptably 4-choosable.
- Triangle free planar graphs are adaptably 3-choosable.
- There are C_4-free planar graphs that are not adaptably 3-colourable.
- For any integer $k \geq 5$, there are planar graphs that are C_t-free for all $5 \leq t \leq k$ and not adaptably 3-colourable.
- For any integer k, there are planar graphs G in which any two triangles have distance at least k and G is not adaptably 3-choosable.

A natural question is whether some combinations of requirements on triangles, 4-cycles and 5-cycles imply adaptable 3-choosability or adaptable 3-colourability. In 1976, Steinberg conjectured that planar graphs without cycles of length 4 and 5 are 3-colourable. There are many partial results on this problem (see [11] for a survey), however, the conjecture itself is still open. The corresponding question for adaptable choosability and adaptable colourability was asked in [2]: Are planar graphs without 4-cycles and 5-cycles adaptably 3-colourable (or even adaptably 3-choosable)?

In this note we consider planar graphs in which no short cycles are close to each other. Suppose A and B are two subgraphs of G. We say A, B are *adjacent* if they have an edge in common. We say A, B are *intersecting* if $V(A) \cap V(B) \neq \emptyset$. The distance between A and B is defined as $d_G(A, B) = \min\{d_G(x, y) : x \in V(A), y \in V(B)\}$.

Theorem 1 *Suppose G is a planar graph. Then G is adaptably 3-choosable if one of the following holds:*

1. *Any two triangles in G do not intersect and no triangle is adjacent to a 4-cycle or a 5-cycle. Moreover, any 6-cycle is adjacent to at most two triangles.*
2. *Any two triangles in G have distance at least 2 and no triangle is adjacent to a 4-cycle.*
3. *Any two triangles in G have distance at least 2 and no triangle is adjacent to a 5-cycle.*

2 Proof of Theorem 1

Suppose G is plane graph and $C = (v_1, \ldots, v_k)$ is the outer face. Let L be a list assignment of permissible colours to vertices of G, such that each vertex x not on C is assigned a set $L(x)$ of 3 permissible colours, and each vertex v_i on C has either 2 permissible colours or 3 permissible colours. An edge $e = xy$ on C is called a *special edge* if $|L(x)| = |L(y)| = 2$. Let $\mathcal{S}$ be the set of special edges of G (with respect to the list assignment L) and let $\mathcal{T}$ be the set of triangles of G and let $\mathcal{C}_i$ be the set of cycles in G of length i. We say L is a *valid list assignment* if one of the following holds:

- If $A, B \in \mathcal{T} \cup \mathcal{S}$ and $A \neq B$, then A, B do not intersect; if $A \in \mathcal{T} \cup \mathcal{S}$ and $B \in \mathcal{C}_4 \cup \mathcal{C}_5$, then A, B are not adjacent; if $A_1, A_2, A_3 \in \mathcal{T} \cup \mathcal{S}$ ($A_i \neq A_j$ if $i \neq j$) and $B \in \mathcal{C}_6$, then there is an $i \in \{1, 2, 3\}$ such that A_i is not adjacent to B.
- If $A, B \in \mathcal{T} \cup \mathcal{S}$ and $A \neq B$, then $d_G(A, B) \geq 2$; if $A \in \mathcal{T} \cup \mathcal{S}$ and $B \in \mathcal{C}_4$, then A, B are not adjacent.
- If $A, B \in \mathcal{T} \cup \mathcal{S}$ and $A \neq B$, then $d_G(A, B) \geq 2$; if $A \in \mathcal{T} \cup \mathcal{S}$ and $B \in \mathcal{C}_5$, then A, B are not adjacent; if $A \in \mathcal{S}$ and $B \in \mathcal{C}_4$, then A, B are not adjacent.

Observe that if a planar graph G has a valid list assignment then a list assignment which assigns to each vertex 3 permissible colours is a valid list assignment. So G has a valid list assignment if and only if G satisfies the condition of Theorem 1.

Lemma 2.1 *If G is plane graph and L is a valid list assignment for G, then for any edge colouring F of G, G has an L-colouring adapted to F.*

Proof. Assume the lemma is not true and G with edge colouring F is a minimum counterexample.

First we prove that G has no separating triangles. Assume to the contrary that $T = \{x, y, z\}$ is a separating triangle. Let G' be obtained from G by deleting the interior of T. Since G is a minimum counterexample, there is an L-colouring c of G' adapted to F. Now we consider the graph $G'' = G - G'$. Let L' be the list assignment defined as follows:

$$L'(v) = \begin{cases} L(v) - \{c(x)\} & \text{if } x \sim v \\ L(v) - \{c(y)\} & \text{if } y \sim v \\ L(v) - \{c(z)\} & \text{if } z \sim v \\ L(v), & \text{otherwise.} \end{cases}$$

Since any two triangles do not intersect, each vertex v in G'' is adjacent to at most one of x, y, z. Moreover, since there is an integer $m \in \{4, 5\}$ such that T is not adjacent to any m-cycle, for any edge uv, if u is adjacent to one of x, y, z, then v is not adjacent to any of x, y, z. Therefore the list assignment L' of G'' has no special edges and hence is a valid list assignment for G''. Hence G'' has an L'-colouring c' adapted to F. Then the union of c and c' is an L-colouring of G adapted to F.

In the following, we assume that G has no separating triangles.

A *chord* of C is an edge v_iv_j such that $v_i, v_j \in C$ and $j \neq i+1 \pmod k$. A *long chord* of C is a path (v_i, x, v_j), connecting two vertices v_i and v_j of C, where $x \notin C$ and $|L(v_i)| + |L(v_j)| = 5$ and $j \neq i+1 \pmod k$. Let $\mathcal{P}$ be the set consisting of all long chords, chords, and cut-vertices on C.

If $P \in \mathcal{P}$ (i.e., P is a chord or a long chord or a cut-vertex on C), then $C - P$ has two components. We denoted by A_p and B_p the two components of $C - P$, and assume that $|A_p| \leq |B_p|$. Let $\mathcal{B}$ be the set of chord P such that $|A_P| = 1$, i.e., P is of the form v_iv_{i+2}. Let $P^* \in \mathcal{P} \setminus \mathcal{B}$ such that $|A_{P^*}|$ is the minimum, i.e., for any $P' \in \mathcal{P} \setminus \mathcal{B}$, $|A(P^*)| \leq |A(P')|$.

Claim A For P^* chosen as above, one of the following holds:

- A_{P^*} contains a triangle.
- A_{P^*} contains a vertex v_t with $|L(v_t)| = 3$ and v_t is not contained in any chord or long chord of C and is not a cut-vertex.

Assume A_{P^*} contains no triangles. We shall show that A_{P^*} contains a vertex v_t with $|L(v_t)| = 3$ and v_t is not contained in any chord or long chord of C and is not a cut-vertex.

We first consider the case that $P^* = (v_i, x, v_j)$ is a long chord. Without loss of generality, we may assume that $|L(v_i)| = 2$ and $|L(v_j)| = 3$ and $A_{P^*} = \{v_{i+1}, v_{i+2}, \cdots, v_{j-1}\}$. If $|A_{P^*}| = 1$, then for $t = i+1$, $|L(v_t)| = 3$, for otherwise, the 4-cycle $(v_i, v_{i+1}, v_{i+2}, x)$ contains a special edge v_iv_{i+1}, in contrary to our assumption. Assume $|A_{P^*}| \geq 2$. Then there is a $t \in \{i+1, i+2\}$ such that $|L(v_t)| = 3$, for otherwise $v_iv_{i+1}, v_{i+1}v_{i+2}$ are two intersecting special edges, in contrary to our assumption.

If v_t is a cut-vertex or v_t is contained in a long chord, then for $P' = v_t$ (in case v_t is a cut-vertex) or P' is a long chord containing v_t, we have $|A_{P'}| < |A_{P^*}|$, in contrary to our choice of P^*. Thus we assume that v_t is not a cut-vertex and is not contained in a long chord. If v_t is not contained in a chord, then we are done. Assume to the contrary that v_t is contained in a chord P'. Since $A_{P'} \subset A_{P^*}$, which implies that $|A_{P'}| < |A_{P^*}|$, by the choice of P^*, we conclude that $P' \in \mathcal{B}$, i.e., $|A_{P'}| = 1$. Assume $P' = v_tv_{t+2}$. Then $T = \{v_t, v_{t+1}, v_{t+2}\}$ induces a triangle. Since no triangle is contained in A_{P^*}, we conclude that T contains a vertex which is not in A_{P^*}. This is possible only if $\{v_t, v_{t+2}\} \cap \{v_i, v_j\} \neq \emptyset$. If $v_t = v_i$, then since $|L(v_i)| = 2$ we must have $|L(v_{i+1})| = 3$, for otherwise, the special edge v_iv_{i+1} intersect the triangle T, in contrary to our assumption. It is obvious that v_{i+1} is not contained in a chord or a long chord and is not cut-vertex.

Assume $v_{t+2} = v_j$. Then $v_t \neq v_{i+1}$, for otherwise G has a 4-cycle $(v_i, v_{i+1}, v_{i+3}, x)$, which is adjacent to the triangle T and G also has a 5-cycle $(v_i, v_{i+1}, v_{i+2}, v_{i+3}, x)$, which is adjacent to the triangle T. If $t \neq i+2$, then there is a vertex $v_{t'} \in \{v_{i+1}, v_{i+2}\}$ such that $|L(v_{t'})| = 3$, for otherwise, there are two intersecting special edges v_iv_{i+1} and $v_{i+1}v_{i+2}$. If $t = i+2$, then for $t' = i+1$, $|L(v_{t'})| = 3$, for otherwise the triangle T has distance 1 to the special edge v_iv_{i+1} and is adjacent to the 5-cycle $(v_i, v_{i+1}, v_{i+2}, v_j, x)$. It is obvious that $v_{t'}$ is not a cut-vertex, not contained in a chord or a long chord.

Next we consider the case that $P^* = v_iv_j$ is a chord and $A_{P^*} = \{v_{i+1}, v_{i+2}, \cdots, v_{j-1}\}$. Since $P^* \notin \mathcal{B}$, we know that $|A_{P^*}| \geq 2$. If $|A_{P^*}| = 2$, then there is a $t \in \{i+1, i+2\}$ such that $|L(v_t)| = 3$, for otherwise, $(v_i, v_{i+1}, v_{i+2}, v_{i+3})$ is a 4-cycle which contains a special edge. If $|A_{P^*}| \geq 3$, then there exist $t \in \{i+1, i+2, i+3\}$ such that $|L(v_t)| = 3$, for otherwise $v_{i+1}v_{i+2}, v_{i+2}v_{i+3}$ are two intersecting special edges, in contrary to our assumption. The same argument as above shows that v_t is not contained in a long chord and is not a cut-vertex. Assume v_t is contained in a chord. Then the same argument as above shows that the chord is of the form v_iv_{i+2} or $v_{j-2}v_j$. Without loss of generality, assume that v_iv_{i+2} is a chord and $v_t = v_{i+2}$. Let T be the triangle induced by $\{v_i, v_{i+1}, v_{i+2}\}$.

Assume first that for any $A, B \in \mathcal{S} \cup \mathcal{T}$, $d_G(A, B) \geq 2$. Then $v_{j-2}v_j$ is not a chord, for otherwise, let $T' = \{v_{j-2}, v_{j-1}, v_j\}$, then the two triangles T and T' have distance at most 1. Since there is an integer $m \in \{4, 5\}$ such that T is not adjacent to any m-cycle, it follows that $v_{i+3}, v_{i+4} \in A_{P^*}$. Then there is a $t' \in \{i+3, i+4\}$ such that $|L(v_{t'})| = 3$, for otherwise the special edge $v_{i+3}v_{i+4}$ has distance 1 to the triangle T. Since $v_{j-2}v_j$ is not a chord, the argument as above shows that $v_{t'}$ is not a cut-vertex, is not contained in a chord or a long chord.

Assume that some $A, B \in \mathcal{S} \cup \mathcal{T}$ have distance less than 2. Then as L is a valid list assignment, we know the following hold:

- For $A, B \in \mathcal{T} \cup \mathcal{S}$ and $A \neq B$, $V(A) \cap V(B) = \emptyset$.
- For $A \in \mathcal{T} \cup \mathcal{S}$ and $B \in \mathcal{C}_4 \cup \mathcal{C}_5$, A, B are not adjacent.
- For $A_1, A_2, A_3 \in \mathcal{T} \cup \mathcal{S}$ ($A_i \neq A_j$ if $i \neq j$) and $B \in \mathcal{C}_6$, there is an $i \in \{1, 2, 3\}$ such that A_i is not adjacent to B.

Since the triangle T is not adjacent to any 4-cycle or 5-cycle, we conclude that $|A_{P^*}| \geq 5$. There is a $t' \in \{i+3, i+4, i+5\}$ such that $|L(v_{t'})| = 3$, for otherwise the special edges $v_{i+3}v_{i+4}, v_{i+4}v_{i+5}$ intersect. The argument as above shows that $v_{t'}$ is not a cut-vertex and is not contained in a long chord. Assume $v_{t'}$ is contained in a chord. By the minimality of A_{P^*}, we know that the chord is of the form $v_{t'}v_{t'+2}$ and $t'+2=j$. This is in contrary to the assumption that L is a valid assignment, because

- If $t'=i+3$, then $(v_i, v_{i+2}, v_{i+3}, v_j)$ is 4-cycle adjacent to a triangle.
- If $t'=i+4$, then $(v_i, v_{i+2}, v_{i+3}, v_{i+4}, v_j)$ is 5-cycle adjacent to a triangle.
- If $t'=i+5$, then $(v_i, v_{i+2}, v_{i+3}, v_{i+4}, v_{i+5}, v_j)$ is 6-cycle adjacent to two triangles and a special edge.

Finally we consider the case that $P^* = v_i$ is a cut-vertex. If $|A_{P^*}| = 2$, then $\{v_i, v_{i+1}, v_{i+2}\}$ induces a triangle. Hence there is a $t \in \{i+1, i+2\}$ such that $|L(v_t)| = 3$, and v_t is not a cut-vertex, not contained in a chord or a long chord. Assume $|A_{P^*}| \geq 3$. Then there exist $t \in \{i+1, i+2, i+3\}$ such that $|L(v_t)| = 3$. The same argument as above shows that v_t is not a cut-vertex, and is not contained in a long chord. Moreover, if v_t is contained in a chord, then by symmetry, we may assume that the chord is v_iv_{i+2}. In this case, $|A_{P^*}| \geq 5$, for otherwise, G have either two intersecting triangles or the triangle T induced by $\{v_i, v_{i+1}, v_{i+2}\}$ is adjacent to a 4-cycle and a 5-cycle, in contrary to our assumption. Then there is a $t' \in \{i+3, i+4\}$ such that $|L(v_{t'})| = 3$, and $v_{t'}$ is not a cut-vertex, not contained in a chord or a long chord. This completes the proof of Claim A. ∎

Case 1 A_{P^*} contains a triangle.

Since the triangle is not separating, the three vertices of the triangle are consecutive vertices of C, say the triangle is $T = \{v_k, v_{k+1}, v_{k+2}\}$, and moreover, v_{k+1} has degree 2. If $|L(v_{k+1})| = 3$, we can color $G - v_{k+1}$ by induction, then color v_{k+1}. Assume $|L(v_{k+1})| = 2$. Then $|L(v_k)| = |L(v_{k+2})| = 3$, otherwise the distances between special edges will less than 3.

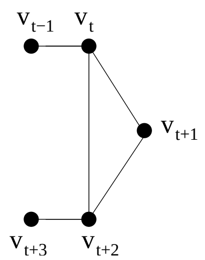

Figure 1: A_{P^*} contains a triangle

Choose a color $\alpha \in L(v_k)$ such that $\alpha \neq F(v_kv_{k-1}), F(v_kv_{k+2})$, a color $\gamma \in L(v_{k+2})$ such that $\gamma \neq F(v_{k+2}v_{k+3}), F(v_{k+2}v_{k+1})$, and a color $\beta \in L(v_{k+1})$ such that $\beta \neq F(v_{k+1}v_k)$. Observe that v_k and v_{k+2} are not contained in any chord or long chord of C, otherwise it will contradict the minimality of A_{P^*}.

Let $G^* = G - \{v_k, v_{k+1}, v_{k+2}\}$. Let L' be the list assignment of G^* obtained from L as follows:

$$L'(x) = \begin{cases} L(x) - \{\alpha\}, & \text{if } x \sim v_k \text{ and } x \neq v_{k-1}, \\ L(x) - \{\gamma\}, & \text{if } x \sim v_{k+2} \text{ and } x \neq v_{k+3}, \\ L(x), & \text{otherwise.} \end{cases}$$

Claim B If $e = xy$ is a special edge of G^*, then e is a special edge of G.

Assume to the contrary that e is not a special edge of G. Since v_k, v_{k+2} are not contained in any triangle other than T, each of v_k, v_{k+2} is adjacent to at most one of x, y. Since there is an integer $m \in \{4, 5\}$ such that T is not adjacent to any m-cycles, at most one of x, y is adjacent to a vertex in v_k, v_{k+2}. As e is a special edge of G^* and not a special edge of G, we conclude that exactly one of x, y is adjacent to one of v_k, v_{k+2}, and the other vertex of x, y has only 2 permissible colours in the list assignment L. By symmetry, we may assume that x is adjacent to v_k and $|L(y)| = 2$. If x is a vertex of C, then $P = v_k x$ is a chord of C. By the choice of P^*, we conclude that $P \in \mathcal{B}$. Hence $|A_P| = 1$ and $A_P \cup P$ induces a triangle T'. But T and T' intersect each other, in contrary to our assumption.

Assume x is not a vertex of C. Then $P' = v_k xy$ is a long chord of C, and $A_{P'}$ and is contained A_{P^*}, in contrary to the choice of P^*. This completes the proof of Claim B. ∎

It follows from the claim above that L' is valid list assignment of G^*. Therefore there is an L'-colouring c of G^* adapted to F. By letting $c(v_k) = \alpha$, $c(v_{k+1}) = \beta$ and $c(v_{k+2}) = \gamma$, we obtain an L-colouring of G adapted to F.

Case 2 A_{P^*} contains no triangle.

By Claim A, there is a vertex $v_t \in A_{P^*}$ such that $|L(v_t)| = 3$ and v_t is not a cut-vertex, not contained in a chord or a long chord. Let $\alpha \in L(v_t)$ such that α is not equal to the colours of the edges $v_{t-1}v_t$ and $v_t v_{t+1}$.

Let $G^* = G - v_t$ and let $\alpha \in L(v_k)$ be a colour such that $\alpha \neq F(v_{t-1}v_t), F(v_t v_{t+1})$. Let L' be the list assignment of G^* obtained from L as follows:

$$L'(x) = \begin{cases} L(x) - \{\alpha\}, & \text{if } x \sim v_t \text{ and } x \neq v_{t-1}, v_{t+1}, \\ L(x), & \text{otherwise.} \end{cases}$$

Since v_t is not contained in a long chord of G, it follows that if $e = xy$ is a special edge of G^*, then either e is a special edge of G or $\{x, y, v_t\}$ induces a triangle. This implies that L' is a valid list assignment of G^*. By induction hypothesis G' has an L'-colouring c adapted to F. We can extend c to an L-colouring of G adapted to F by colouring v_t with colour α. ∎

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